Table 1 Eigenvalues for clamped sandwich plate, $a/c = 5$ and $\lambda^{*2} = \omega a^2(\rho/D)^{1/2}$

	$\phi = b/a$								
Mode	1.0	1.25	1.5	1.75	2.0	2.25	2.5	2.75	3.0
1	32.32	27.33	24.94	23.66	22.92	22.46	22.15	21.94	21.78
2	58.51	44.25	36.31	31.31	28.68	27.77	25.48	24.57	23.92
3	78.51	55.36	53.38	44.35	38.36	34.26	31.37	29.29	27.75
4	90.58	67.56	62.11	58.84	50.99	44.39	39.59	36.02	33.32
5	125.34	80.34	73.64	60.25	52.46	52.12	49.60	44.42	40.41

Table 2 Eigenvalues for clamped sandwich plate, $a/c = 15.8$ and $\lambda^{*2} = \omega a^2(\rho/D)^{1/2}$

	$\phi = b/a$								
Mode	1.0	1.25	1.5	1.75	2.0	2.25	2.5	2.75	3.0
1	35.59	29.62	26.79	25.27	24.41	23.86	23.49	23.24	23.06
2	71.44	51.51	41.08	35.13	31.49	29.14	27.56	26.46	25.66
3	103.75	66.81	64.56	52.08	44.01	38.66	34.96	32.34	30.42
4	124.96	86.17	77.39	63.30	61.72	52.38	45.82	41.07	37.55
5	154.51	118.25	96.76	72.33	69.74	62.02	59.94	52.57	47.04

Table 3 Comparison of natural frequencies of clamped sandwich plate: $\omega = (\lambda^*/a)^2(D/\rho)^{1/2}$, $t = 0.4064$ mm, $c = 6.35$ mm, $a = 1016$ mm, $G_c = 6.895 \times 10^6$ Pa, $E_f = 6.896 \times 10^{10}$ Pa, $\rho = 5343.4$ Ns²/m⁴

	$\phi = 1.0$		$\phi = 1.5$		$\phi = 2.0$	
Mode	Present solution	Ref. 4	Present solution	Ref. 4	Present solution	Ref. 4
1	68.403	68.356	53.446	53.410	49.360	49.327
2	118.443	118.381	75.856	75.807	60.798	60.760
3	154.802	154.717	108.322	108.434	79.634	79.672

little change in the eigenvalues or mode shapes beyond 10 terms. It was, therefore, concluded that good convergence is obtained with 10 terms and all of the calculations presented in this paper are with $k = 10$.

To demonstrate the accuracy of the method comparisons between the data computed here and those given in Ref. 4 are made in Table 3. Results in Ref. 4 are based on Galerkin's method. It is found that there is good agreement between the computed results. Although only one example is shown here, it is obvious that other general boundary conditions can be handled the same way. They would lead to a new matrix $[C]$ in Eq. (15).

The approach is simple and quite efficient as the displacement solution is obtained in terms of definite integral, which are functions of boundary constraints and the corresponding displacement functions.

References

[1]Reissner, E., "Finite Deflections of Sandwich Plates," *Journal of the Aerospace Sciences*, Vol. 15, 1948, pp. 435–440.

[2]Falgout, T. E., "A Differential Equation of Free Transverse Vibration of Isotropic Sandwich Plate," *Proceedings of the 7th Mid-Western Mechanic Conference*, Vol. 1, 1961.

[3]Ahmed, K. M., "Vibration Analysis of Doubly Curved Honeycomb Sandwich Plates by the Finite Element Method," Inst. of Sound and Vibration Research, TR 37, Univ. of Southampton, Sept. 1970.

[4]Ng, S. S. F., and Das, B., "Free Vibration and Buckling Analysis of Clamped Skew Sandwich Plates by the Calerkin Method," *Journal of Sound and Vibration*, Vol. 107, No. 1, 1986, pp. 97–106.

[5]Fu, B., and Li, N., "The Method of the Reciprocal Theorem of Forced Vibration for the Elastic Thin Rectangular Plates (I)—Rectangular Plates with Four Clamped Edges and with Three Clamped Edges," *Journal of Applied Mathematics and Mechanics*, Vol. 10, No. 8, 1989, pp. 727–749.

[6]Li, N., "The Reciprocal Theorem Method for Plate Free Vibration Analysis—The Completely Free Rectangular Plate," *Journal of Sound and Vibration*, Vol. 156, No. 2, 1992, pp. 357–364.

New Evolutionary Direction Operator for Genetic Algorithms

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I. Introduction

GENETIC algorithms are search algorithms based on a natural evolution mechanism.[1] Recently, some aerodynamic optimization problems have been solved by genetic algorithms,[2–5] because genetic algorithms are robust and suitable for finding optimum effectively with a smaller possibility of falling in local optima than other algorithms. Gradient-based optimizers are another way to solve optimization problems. They are able to find a local optimum efficiently but are not superior to genetic algorithms from the viewpoint of global optimization.

To find global optima efficiently, the introduction of "evolutionary direction" into genetic algorithms is expected to be useful. A variety of methods to determine the evolutionary direction may be considered. One of them is a gradient-based optimizer.[6] However, gradient-based optimizers are usually time consuming because evaluation of gradients is necessary. In this paper, we propose a new operator for genetic algorithms to determine the evolutionary direction in a simple way. The operator does not require evaluation of gradients and thus is much less time consuming than the gradient-based optimizers.

II. Evolutionary Direction Operator

An optimization procedure by means of a genetic algorithm with an evolutionary direction is shown in Fig. 1. First, we prescribe the population size and select individuals of the population randomly (initialization). Then we evaluate the chromosomes and obtain fitness of each individual according to an evaluation function that is given a priori. In the case of optimization of aerodynamic configuration, for example, drag and/or lift evaluated by a computational

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fluid dynamics (CFD) solver are often used as fitness. If the result of the evaluation is not satisfactory, then the evolution process is performed. Three operators are used for evolution in each generation: the selection and crossover operator, the mutation operator, and the evolutionary direction operator. The selection and crossover operator and mutation operator are used for genetic algorithms, whereas the evolutionary direction operator is used to determine the evolutionary direction. Here the term "operator" means a specific technique to reproduce children from their parents. Each operator has its own rate of reproduction. In Fig. 1, Pc, Pm, and Ped are the rates at which children are reproduced by the selection and crossover, mutation, and evolutionary direction operators, respectively. For example, when the population size is 100 and Pc is 0.6, 60 individuals are reproduced by the selection and crossover operator. The selection and crossover operator and the mutation operator used in this paper are conventional ones.[1] The evolutionary direction operator that we propose in this paper is described as follows.

Evolution from a present chromosome to the next generation chromosome is assumed to be determined from chromosomes and fitnesses of both parents and the present. According to gray scale coding,[6] a chromosome is assumed to be represented by n nonnegative integers C_p $(p = 1, \ldots, n)$:

$$\text{Chromosome: } \{C_1, C_2, \ldots, C_n\} \tag{1}$$

Each integer represents a design parameter such as, for example, thickness and shape of the mean camber line of a wing in an aerodynamic optimization problem of wing shape. The value of C_p is assumed to range from 0 to $CMAX$:

$$C_p \in [0, CMAX], \qquad p = 1, 2, \ldots, n \tag{2}$$

Figure 2 shows schematically the evolution of an individual determined by the evolutionary direction operator in a two-dimensional optimization problem. The coordinates C_1 and C_2 are the design parameters to be optimized. The third coordinate is the fitness. With

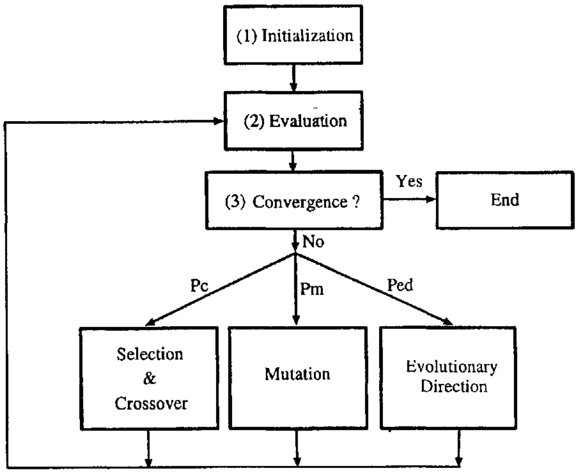


Fig. 1 Optimization procedure.

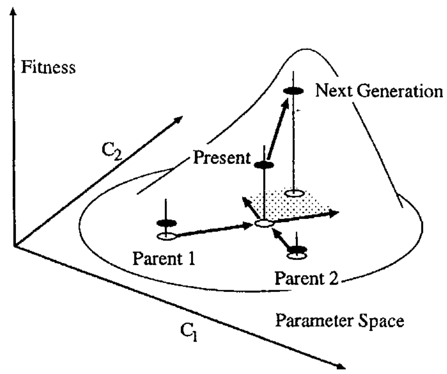


Fig. 2 Schematic of evolutionary direction operator.

the use of the evolutionary direction operator, the chromosome of the next generation, i.e., child, is determined from the chromosomes and the fitnesses of present and parents as follows:

Inputs:

$$\text{Parent 1: Chromosome: } \{CP1_1, CP1_2, \ldots, CP1_n\}$$

$$\text{Fitness: } F_{p1}$$

$$\text{Parent 2: Chromosome: } \{CP2_1, CP2_2, \ldots, CP2_n\}$$

$$\text{Fitness: } F_{p2}$$

$$\text{Present: Chromosome: } \{C_1, C_2, \ldots, C_n\}$$

$$\text{Fitness: } F$$

Output:

$$\text{Child: Chromosome: } \{CC_1, CC_2, \ldots, CC_n\}$$

In the preceding expressions,

$$CC_p = \max\left[\min\left(C_p^*, \mathrm{CMAX}\right), 0\right], \qquad p = 1, 2, \ldots, n \tag{3}$$

$$C_p^* = C_p + S \cdot \operatorname{sign}(F - F_{p1}) \cdot (C_p - CP1_p) + T \cdot \operatorname{sign}(F - F_{p2}) \cdot (C_p - CP2_p) \tag{4}$$

$$S, T \in [0, 1] \tag{5}$$

As seen from Fig. 2, we have two directions of the evolution: one from parent 1 to the present and another from parent 2 to the present. The evolution to the next generation is determined from these two directions by using Eqs. (3–5). The contribution of the direction from parent 1 is expressed by the second term of the right-hand side of Eq. (4), whereas that of the direction from parent 2 is by the third term. The random floating point numbers S and T give the magnitude of the contributions of the two directions to the evolution. The signs of the directions are selected depending on whether or not the present fitness was improved through the evolution from the parents. As all of the variables (C_p, $CP1_p$, $CP2_p$, F, F_{p1}, F_{p2}) in Eqs. (3–5) are known, the chromosome of the child CC_p is easily determined. Therefore, the computation time is much less than that by gradient-based optimizers that require evaluation of gradients. Fitness of the child is determined from the child's chromosome after CC_p ($p = 1, \ldots, n$) are obtained. The evolution and evaluation processes are repeated until the evaluation gives satisfactory results (convergence).

III. Maximum Value Search Problem

We evaluate the performance of a genetic algorithm with the evolutionary direction operator by solving a maximum value search problem of a two-dimensional multiple-peak function and by comparing the results with those by traditional genetic algorithms without the evolutionary direction operator. When a maximum value search problem is solved by gradient-based optimizers, it is known that the solutions often fall in a local optimum rapidly.[1] Our purpose here is to demonstrate that by solving this problem with the evolutionary direction operator, the solution reaches the global optimum without falling in a local optimum and computation is more effective than that of the genetic algorithm without the evolutionary direction operator. In this study, we define a two-dimensional multiple-peak function $F(x, y)$ as superposition of normal distributions as follows:

$$F(x, y) = \sum_i^n H_i \cdot \exp\left[-\frac{(x - x_i)^2 + (y - y_i)^2}{2\sigma_i^2}\right] \tag{6}$$

$$H_i \in [H_{\min}, H_{\max}], \qquad \sigma_i \in [\sigma_{\min}, \sigma_{\max}] \tag{7}$$

$$x_i \in [x_{\min}, x_{\max}], \qquad y_i \in [y_{\min}, y_{\max}] \tag{8}$$

where

$$
\begin{aligned}
&n = 50\\
&H_{\min} = 0.0, \qquad H_{\max} = 1.0\\
&\sigma_{\min} = 0.3, \qquad \sigma_{\max} = 1.3\\
&x_{\min} = -10.0, \qquad x_{\max} = 10.0\\
&y_{\min} = -10.0, \qquad y_{\max} = 10.0
\end{aligned} \tag{9}
$$

The preceding values of parameters H_i, σ_i, x_i, and y_i were selected arbitrarily. Contour lines of the two-dimensional multiple-peak function selected are shown in Fig. 3. The maximum value of the function is 1.890 at $(x, y) = (-1.62, 0.71)$. In solving this problem, we use two nonnegative integers, C_1 and C_2, to express the arguments of the function:

$$\text{Chromosome: } \{C_1, C_2\} \tag{10}$$

The variables (x, y) in Eq. (6) are obtained from C_1 and C_2 by decoding a chromosome to real numbers as follows:

$$x = (C_1/CMAX - 0.5)(x_{\max} - x_{\min}) \tag{11}$$

$$y = (C_2/CMAX - 0.5)(y_{\max} - y_{\min}) \tag{12}$$

The value $CMAX$ was prescribed to be $2^{31} - 1$ because the computer used in this calculation was a 32-bit machine. Once x and y are known, fitness $F(x, y)$ is determined by Eq. (6). The initial values of C_1 and C_2 were prescribed randomly. In this computation, when C_p^* in Eq. (4) became out of range expressed by Eq. (2), the value was clipped in the range according to Eq. (3).

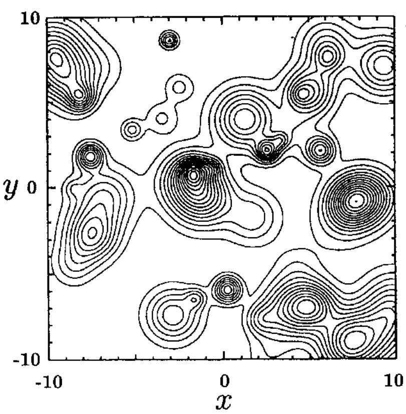


Fig. 3 Two-dimensional multiple-peak function.

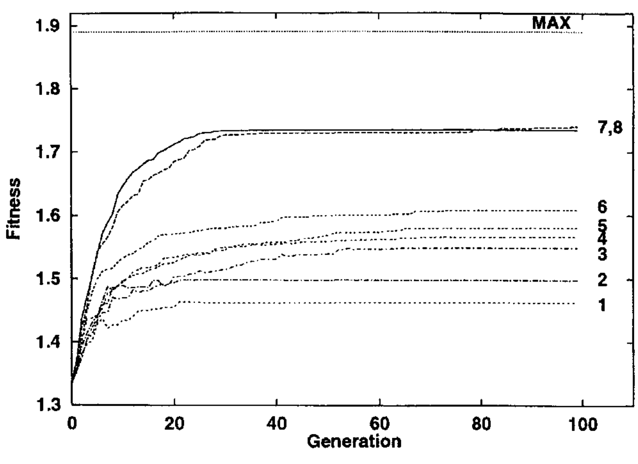
IV. Numerical Experiment and Discussions

To examine the performance of the evolutionary direction operator, and also the effects of elitist strategy and parameters (Pc, Pm, Ped), we solved the maximum value search problem for eight different strategies of genetic algorithms shown in Table 1. The strategies 1–6 are for the cases of traditional genetic algorithms with different rates of crossover and mutation and with or without the use of elitist strategy. The strategies 7 and 8 are for the cases of a genetic algorithm with the evolutionary direction operator with different rates of evolutionary direction.

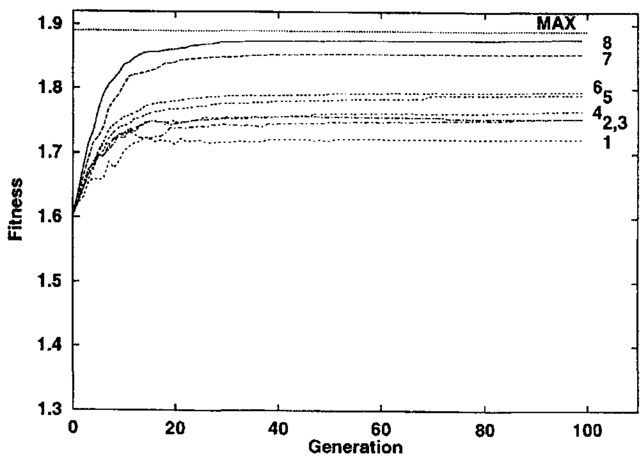
The maximum value search problem was solved for the two cases of population sizes: 50 (case a) and 200 (case b). The results are shown in Figs. 4a and 4b. The lines in the figure denote the maximum fitnesses, respectively, corresponding to a strategy presented in the Table 1. They are averaged over 40 trials for different initial values of C_1 and C_2. The initial values for the 40 trials are the same for all of the 8 strategies. The symbol MAX (= 1.890) in the figures indicates the theoretical maximum value of the function $F(x, y)$ in Eq. (6). Figure 4a shows that the maximum fitness approaches closer to MAX with increasing rates of crossover and mutation. By comparing the results of strategies 5 and 6 with those of strategies 3 and 4, we note that elitist strategy also have an effect on the maximum fitness. As seen from Fig. 4a, strategies 7 and 8 show much better results than other strategies, indicating the effectiveness of the evolutionary direction operator proposed in this paper. Figure 4b shows convergence history in the case of the population size 200. The results show the same tendency of the convergence history as in the case of population size 50 shown in Fig. 4a. Figure 4b also shows that with increasing population size the convergence rate of fitness against the generation number is improved. By comparing Figs. 4a and 4b, the performance of strategy 8 in the case of population size 50 is almost equivalent to that of the genetic algorithms without the evolutionary direction operator (strategies 1 to 6) in the case of population size 200.

Table 1 Strategies of genetic algorithms

Strategy no.	Crossover rate, Pc	Mutation rate, Pm	Evolutionary direction rate, Ped	Elitist strategy
1	0.5	0.0	——	No
2	0.8	0.0	——	No
3	0.5	0.01	——	No
4	0.8	0.01	——	No
5	0.5	0.01	——	Yes
6	0.8	0.01	——	Yes
7	0.5	0.01	0.2	Yes
8	0.5	0.01	0.4	Yes



a) Population size 50



b) Population size 200

Fig. 4 Convergence history: strategy 1 - - - - - -, strategy 2 — - — - — -, strategy 3 — - — - —, strategy 4 - - - - - - -, strategy 5 - - - - - - -, strategy 6 - - - - - - -, strategy 7 — — — — — -, and strategy 8 ———.

V. Conclusions

We propose a new evolutionary direction operator for genetic algorithms. We solved a maximum value search problem of a two-dimensional multiple-peak function. The results show that the evolutionary direction operator is very effective. According to this operator, the evolutionary direction is determined simply, and thus computation time is reduced if compared with the gradient-based optimizers that require evaluation of gradients. This operator is expected to be applicable to almost every genetic algorithm easily because details of the target problem are not required.

References

[1]Goldberg, D. E., *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison–Wesley, Reading, MA, 1989.

[2]Mosetti, G., and Poloni, C., *Aerodynamic Shape Optimization by Means of a Genetic Algorithm, Proceedings of the 5th International Symposium on Computational Fluid Dynamics-Sendai*, Vol. 2, 1993, pp. 279–284.

[3]Quagliarella, D., and Cioppa, A. D., "Genetic Algorithms Applied to the Aerodynamic Design of Transonic Airfoils," AIAA Paper 94-1896, 1994.

[4]Gage, P., and Kroo, I., "A Role for Genetic Algorithms in a Preliminary Design Environment," AIAA Paper 93-3933, Aug. 1993.

[5]Crispin, Y., "Aircraft Conceptual Optimization Using Simulated Evolution," AIAA Paper 94-0092, Jan. 1994.

[6]Davis, L., *Handbook of Genetic Algorithms*, Van Nostrand Reinhold, 1990.

Quick Optimum Buckling Design of Axially Compressed, Fiber Composite Cylindrical Shells

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Nomenclature

A = membrane stiffness matrix
B = coupling stiffness matrix
D = bending stiffness matrix
M = moment vector per unit length
m = axial half-wave number of the deformation mode
N = force vector per unit length
n = circumferential wave number of the deformation mode
ε = reference surface strain vector
κ = curvature change vector

Introduction

LIGHTWEIGHT, thin-walled structures loaded by compression and/or shear may fail due to buckling. Laminates made from fiber-reinforced plastics (FRP) are promising candidates for use in such applications. Among them, axially compressed FRP cylindrical shells play an important role, both because of their application as real shells and because of their suitability to serve as specimens for basic buckling research. Such shells could be accomplished as stiffened, unstiffened, or sandwich shells. Which of these types will be chosen depends on real application. The decision to be made requires a comparison on the basis of optimum respective configurations. Thus, optimization of the different types of construction is a precondition for the choice of the best type, and quick optimum design is desirable. The present work considers unstiffened shells. As the failure is dominated by buckling, which on the other hand is strongly affected by laminate stiffnesses, laminate stacking is of main influence; thus, this should be considered in optimum design. But, in particular, fiber orientations as design variables require substantial expenditure of computer time, which impedes quick design. The application of simple rules might be the remedy to overcome this conflict. It is the main objective of this work to look for such simple rules for quick design. They will be derived from a database, which is generated by a great many of numerical optimization runs. The following sections on the structural model and optimization procedure describe how this database has been generated, whereas the results section reports on its evaluation leading to the simple rules for quick optimum design. The literature available, e.g., Refs. 1–3, merely considers subaspects of the problem. The work presented here is part of a much more extended study.[4,5]

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Structural Model

Figure 1 shows the cylinder considered, with radius R, length L, and compressive force F. The wall of the shell consists of layers with thicknesses $t_1, t_2, \ldots, t_p$, where the index 1 denotes the inner layer and the index p the outer one. The thicknesses of the layers are constant over the length. The thinnest unit is a tape, which consists of unidirectionally oriented fibers embedded in a matrix. The next thickest unit is an angle ply, formed from two tapes: one is arranged at $+\alpha_i$ to the axis of the shell and the other one at $-\alpha_i$. The index i denotes the layer to which the tapes belong. The thickness of this unit is t_0; the unit is called here basic unit. From now on, $+\alpha_i$ and $-\alpha_i$ will be considered as a single orientation α_i. Several of the basic units are coupled together resulting in the whole laminate, the thickness of which is $t_l = p \cdot t_0$. The parameter p is the number of basic units in the laminate and, as such, the maximum possible number of layers with different orientations α_i. Apart from considering an angle ply to be homogeneous over its thickness, no simplification is made in regard to stacking. In particular, symmetry of the entire laminate is not postulated. The mechanical characteristics of this setup are described by means of the stiffness matrix of the laminate, which is formulated assuming classical lamination theory,[6]

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa \end{bmatrix} \tag{1}$$

The materials considered here are carbon fiber-reinforced plastics (CFRP) with $Q_{11} = 124446$ N/mm^2, $Q_{12} = 2802$ N/mm^2, $Q_{22} = 8771$ N/mm^2, and $Q_{66} = 5695$ N/mm^2 as stiffness properties of a unidirectional layer, and glass fiber-reinforced plastics (GFRP) with $Q_{11} = 47120$ N/mm^2, $Q_{12} = 3862$ N/mm^2, $Q_{22} = 13316$ N/mm^2, and $Q_{66} = 4300$ N/mm^2.

The model behind the buckling load analysis must allow fast computations, but it also should keep the main attributes affecting buckling. Thus, the classical bifurcation buckling formula for axially compressed, eccentrically orthotropic, shallow cylindrical shells will be used,[7] which can be written as follows:

$$\tilde{F} = 2\pi R \left\{ \min_{m,n} \left[\frac{1}{\beta^2} \left(S_1 + \frac{S_2^2}{S_3} \right) \right] \right\} \tag{2}$$

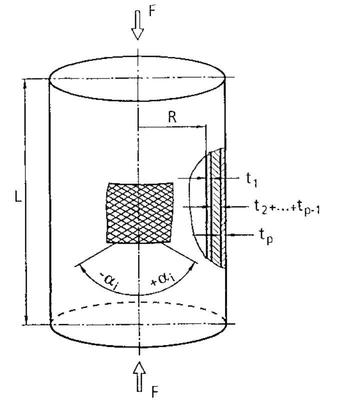

Fig. 1 Axially compressed FRP cylindrical shell.